IBL approaches to Geometry and Probability for High School Teachers

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Legacy of R.L. Moore Conference
Austin, Texas
April 12, 2007

Outline

1. Rationale
   - Instructors' background
   - Goals
2. Implementation
   - Geometry
     - Theme
     - Class Details
   - Probability
3. Evaluation
   - Questions
   - Results
4. Self-evaluation and conclusions

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Bret's background

What living in Madison can do to you

- Graduate work was in finite group theory
- Minored in math education
- KTI Program
- Core Plus and Connected Mathematics Project (CMP)

Matt's background

How on Earth did I get so jaded?

- Geometer by training, teacher by trade
- Third time through a probability course for teachers
- First time: team taught, disconnected
- Second time: interesting for me, over their head
- Third time: ???

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Goals for Math E-302

"Math for Teaching Geometry"

- Maximize student learning
- Improve communication skills
- Motivate students
- Provide a classroom model

Goals for Math E-304

"Inquiries into Probability and Combinatorics"

- Build a discipline from the ground up
- Teach students what they're ready to learn
- Develop ability to read, write, and criticize mathematical arguments

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Platform for inquiry

- Taxicab geometry

Platform for inquiry

- Taxicab geometry
- Compare and contrast with Euclidean

Class Format

- Meet once per week

Class Format

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- Class length is two hours

Class Format

- Meet once per week
- Class length is two hours
- Mostly in-service high school teachers

Role of Instructor

- Moderate discussion

Role of Instructor

- Moderate discussion
- Referee

Role of Instructor

- Moderate discussion
- Referee
- Ask questions

Role of Instructor

- Moderate discussion
- Referee
- Ask questions
- Not an authority

A typical day

- Review

A typical day

- Review
- Work on one problem

A typical day

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- Work on one problem
- 10% lecture

A typical day

- Review
- Work on one problem
- 10% lecture
- 45% small group work

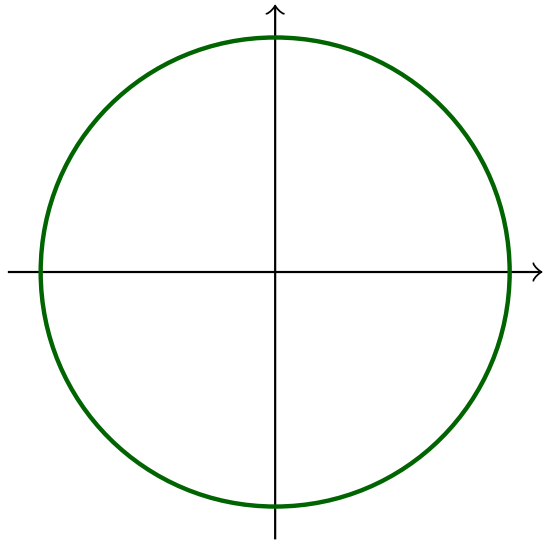
A typical day

- Review
- Work on one problem
- 10% lecture
- 45% small group work
- 45% large group discussion

A typical problem

A typical problem

- What is the definition of a circle in Euclidean geometry?



A typical problem

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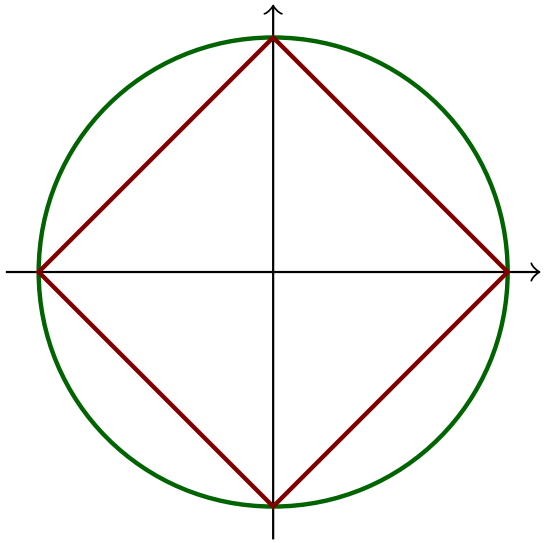
- What does a circle look like in taxicab geometry?



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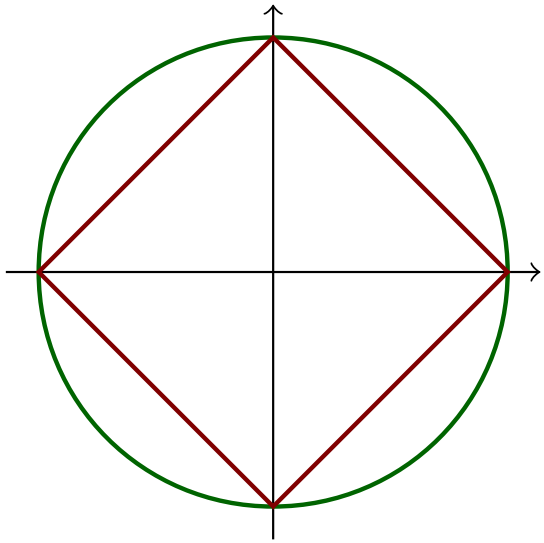

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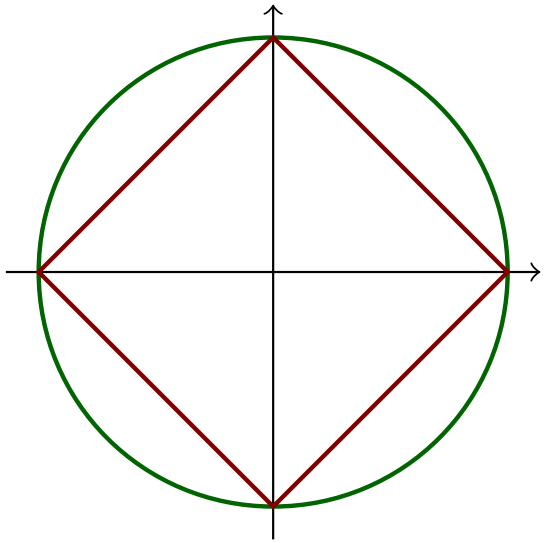
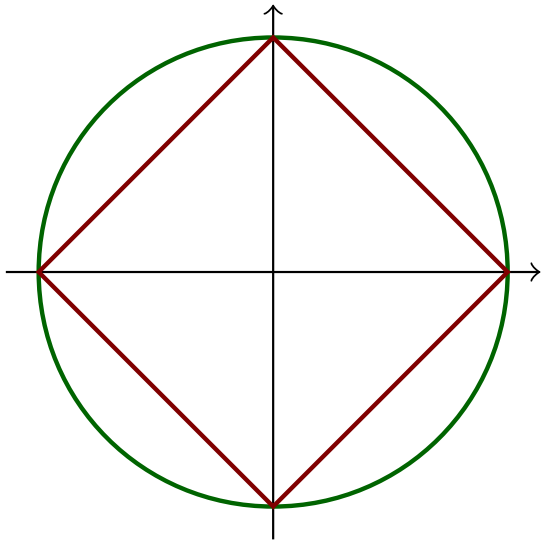
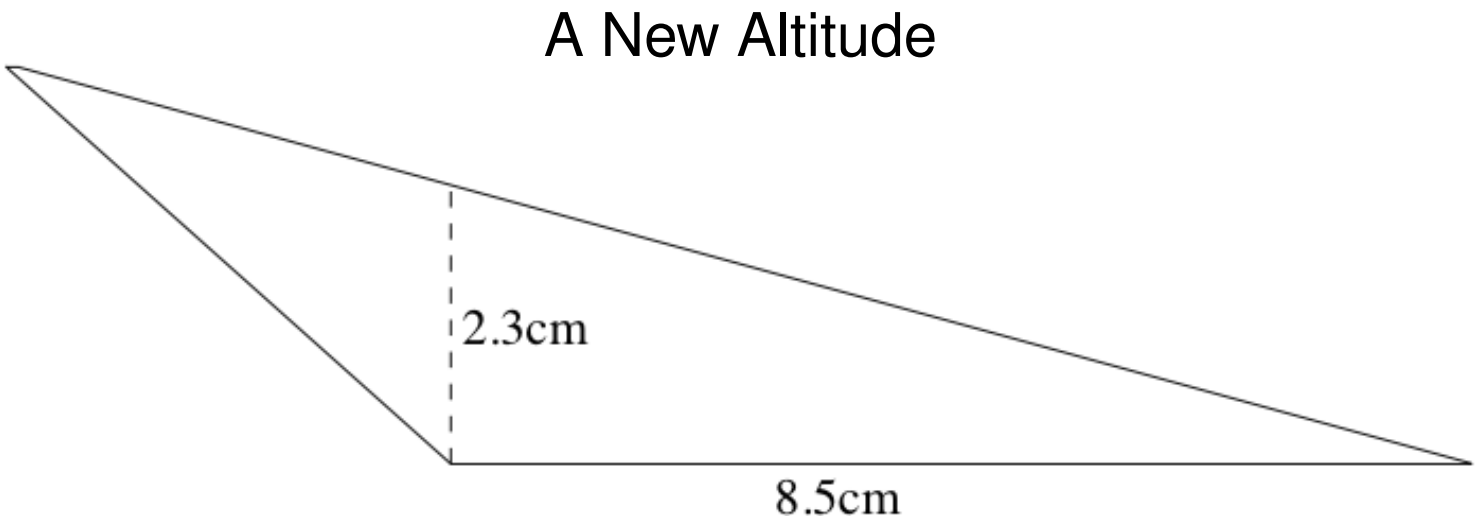


A typical problem

- What is the definition of a circle in Euclidean geometry?
- What does a circle look like in taxicab geometry?
- What is the diameter of a circle in taxicab geometry?
- What is the circumference in taxicab geometry?
- What is π in taxicab geometry?



Another example



$$A = \frac{1}{2}(2.3)(8.5) = 9.775$$

Grading

- Mostly papers

Grading

- Mostly papers
- Two exams

Grading

- Mostly papers
- Two exams
- Class participation

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Probability course is TMM

- I write problems

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- I write problems
- Students submit written up problems

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- I write problems
- Students submit written up problems
- Students present solutions

Probability course is TMM

- I write problems
- Students submit written up problems
- Students present solutions
- I update notes with solutions

Notes TOC

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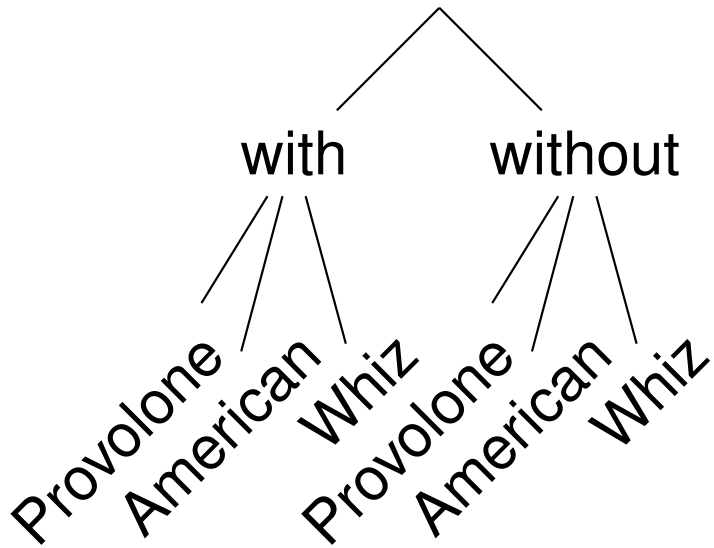
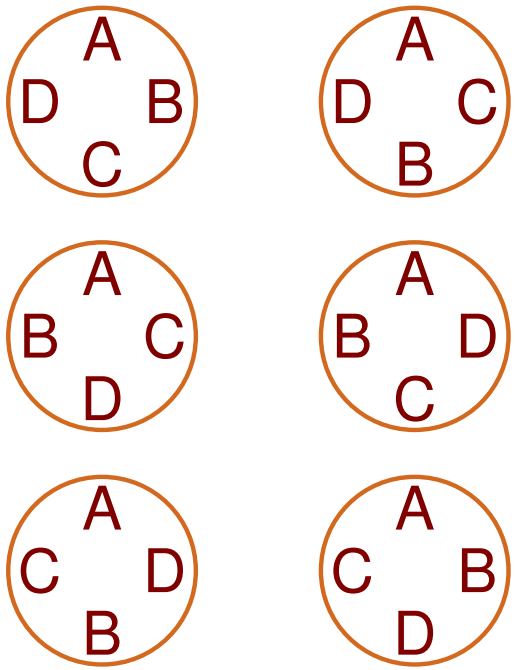
- The Fundamental Counting Principle



Notes TOC

- The Fundamental Counting Principle
- Permutations



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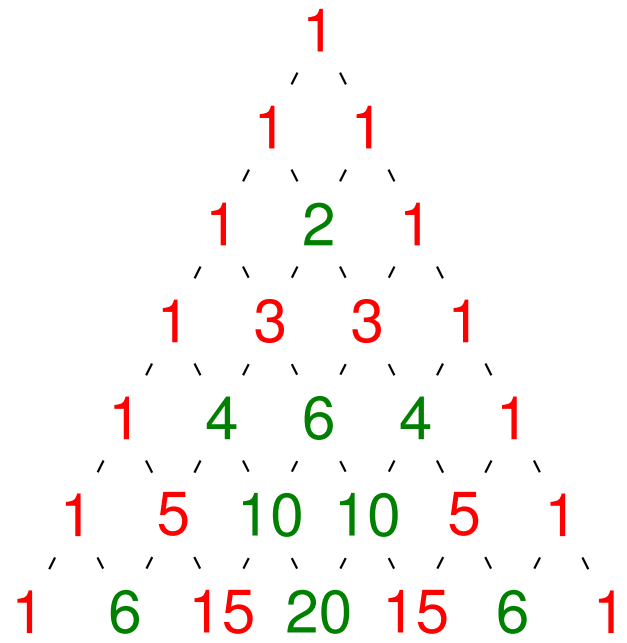
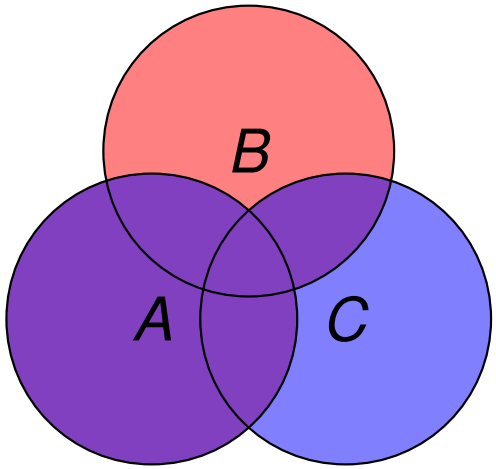
- Combinations



Notes TOC

- The Fundamental Counting Principle
- Permutations
- Combinations
- Set theory



$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Notes TOC

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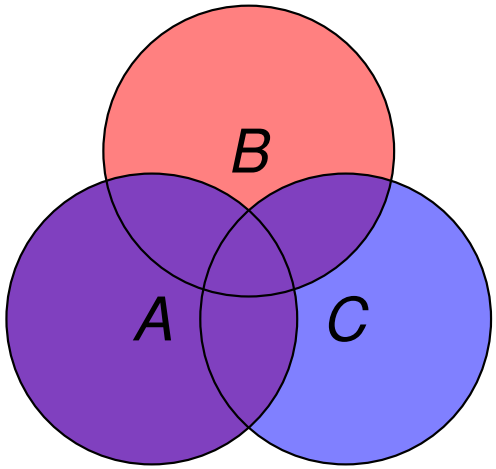
- Permutations
- Combinations
- Set theory
- Axioms of probability



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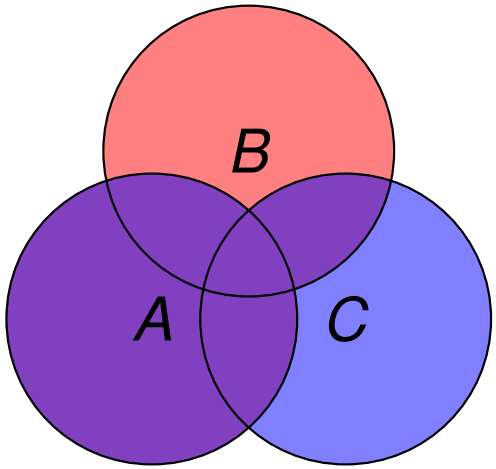
- Expected value



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- . . .

Fun problems

- Give them a menu; ask how many combination plates can be ordered

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- Verify the published probabilities for winning various lottery games

Fun problems

- Give them a menu; ask how many combination plates can be ordered
- Verify the published probabilities for winning various lottery games
- Why can we multiply probabilities of "consecutive" events?

A typical day

- I will have assigned a chapter's worth of problems

A typical day

- I will have assigned a chapter's worth of problems
- I solicit volunteers to present

A typical day

- I will have assigned a chapter's worth of problems
- I solicit volunteers to present
- We watch and question the presenters

A typical day

- I will have assigned a chapter's worth of problems
- I solicit volunteers to present
- We watch and question the presenters
- I stay seated

Grading

- $\geq$ 1 problem written per week, 0-4 scale

Grading

- ≥ 1 problem written per week, 0-4 scale
- ≥ 1 problem presented per week, 0-4 scale

Grading

- ≥ 1 problem written per week, 0-4 scale
- ≥ 1 problem presented per week, 0-4 scale
- Take-home final to come

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Questions

- Influence thinking, teaching, or communicating?

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- Learn more than traditional format?

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- Influence thinking, teaching, or communicating?
- Learn more than traditional format?
- Challenging? Rewarding?

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- Take another class?

Questions

- Influence thinking, teaching, or communicating?
- Learn more than traditional format?
- Challenging? Rewarding?
- Take another class?
- Recommend class format?

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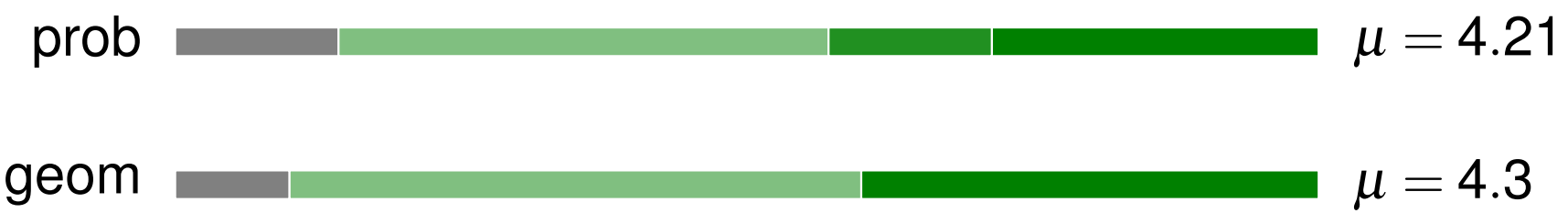
The results: Question 1

How has this course affected the way you think about mathematics?

- 5=Very positively
- 4=Somewhat positively
- 3=No change
- 2=Somewhat negatively
- 1=Very negatively



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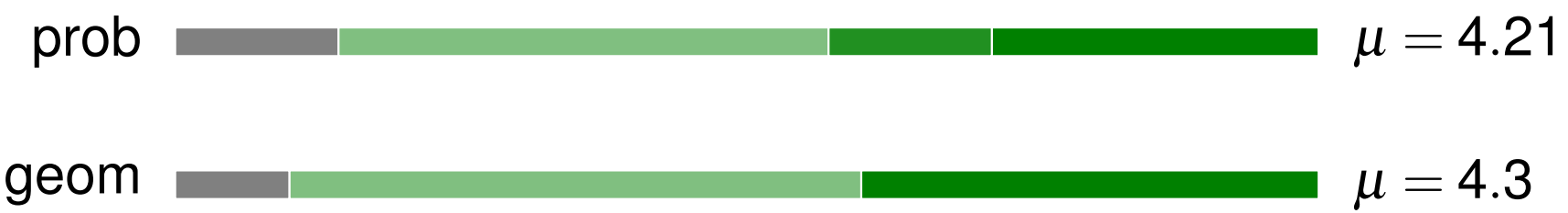


Question 2

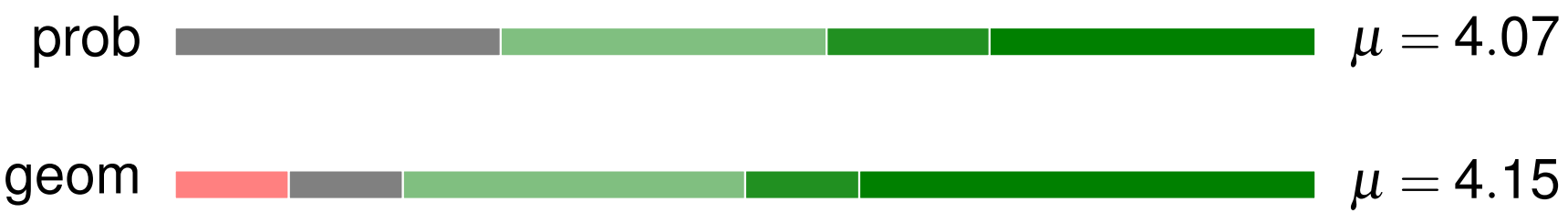
How has this course affected the way you think about **teaching** mathematics?

- 5=Very positively
- 4=Somewhat positively
- 3=No change
- 2=Somewhat negatively
- 1=Very negatively

Question 3

How has this course affected the way you think about **communicating** in mathematics?

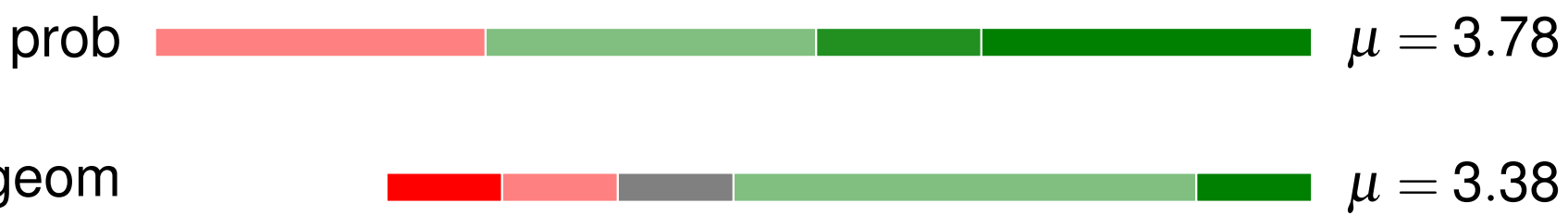
- 5=Very positively
- 4=Somewhat positively
- 3=No change
- 2=Somewhat negatively
- 1=Very negatively



Question 4

Do you think that you learned more, less, or as much as you would have in a more traditionally taught course?

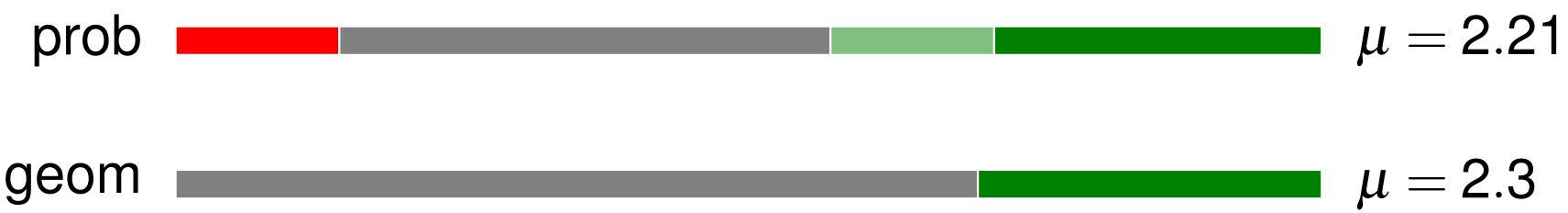
- 5=Much, much more
- 4=A little more than usual
- 3=No change in learning
- 2=A little less than usual
- 1=A lot less than usual



Question 5

How challenging is this course?

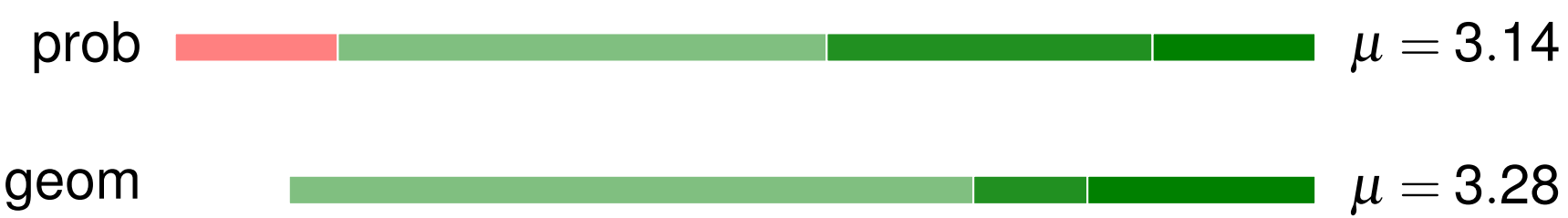
- 3=Very challenging. I had to think much harder than I normally do.
- 2=Sort of challenging.
- 1=Not challenging at all. I could do this in my sleep.



Question 6

How rewarding is this course?

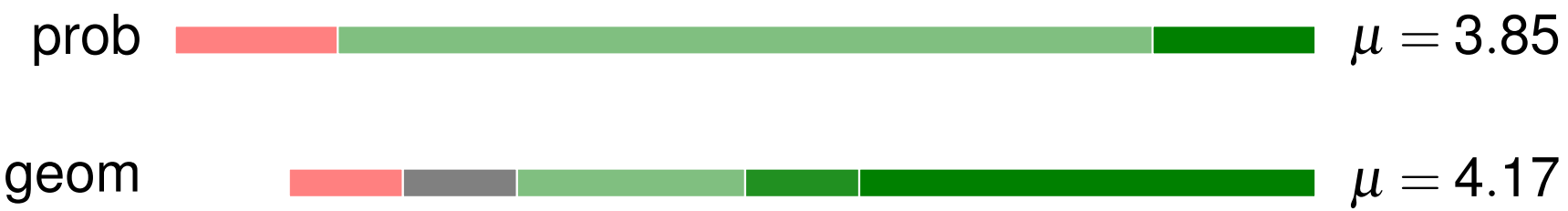
- 4=Ridiculously rewarding. Math is more fun than watching *Dancing with the Stars*!
- 3=Sort of rewarding
- 2=I don't get anything out of it
- 1=I feel like this class saps my will to live.



Question 7

Would you like to take another course taught in this format?

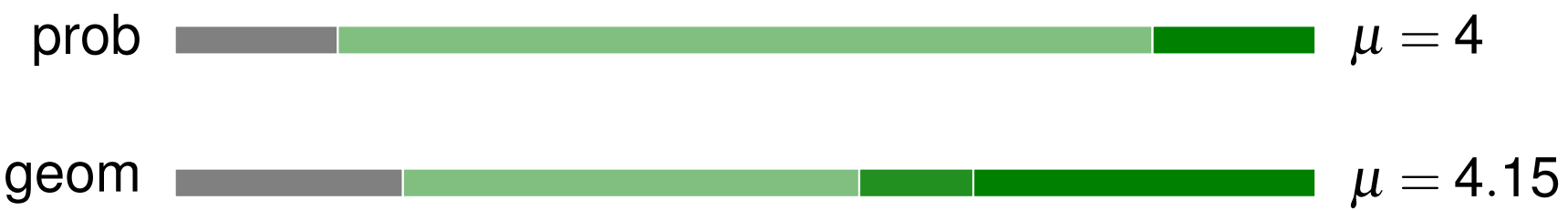
- 5=Yes! Where do I sign up?!?
- 4=Yes, with some reservation
- 3=Undecided
- 2=No
- 1=Hell no



Question 8

Would you recommend a course taught in this format?

- 5=Yes! I want to share the love!
- 4=Sure, it was pretty good.
- 3=Undecided
- 2=No.
- 1=Yes, but only to my worst enemy.



Some quotes from the probability class

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- “Waiting for the other students to finish is a bit of a waste of time.”
- “I don’t necessarily like the experience, but at least it was pedagogically interesting.”

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- “It is tiring to think this hard consistently, but good still.”
- “I wish there was more concrete learning.”
- “I leave excited and bewildered.”

Our own reactions: Probability

- Teaching TMM is harder than lecturing

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- When “drama” happens outside of class, class can be boring

Our own reactions: Probability

- Teaching TMM is harder than lecturing
- When “drama” happens outside of class, class can be boring
- weakest students are involved

Our own reactions: Geometry

- IBL is more difficult to teach

Our own reactions: Geometry

- IBL is more difficult to teach
- IBL is more rewarding to teach

Our own reactions: Geometry

- IBL is more difficult to teach
- IBL is more rewarding to teach
- Still difficult to keep weaker students involved