Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Non-commutative Ring Theory

- Give an example of a simple ring. [**Bergman**]
- Give an example of a simple ring which is not a field. [**Bergman**]
- Give an example of a simple ring which is not a field, and that doesn't have dimension 4 over its center. [**Bergman**]
- Find the center of the Weyl Algebra. [**Bergman**] (The Weyl Algebra was given as an answer to (3) above)
- What are the minimal 1-sided ideals of the Weyl algebra? [**Lam**]
- How could you use Rieffel's Theorem to prove (5)? [**Lam**]
- Define von Neumann regular rings. What are the commutative von Neumann regular rings? [**Lam**]
- Give an example of a commutative von Neumann regular ring that is not a direct product of fields. [**Bergman**]
- What about the Krull dimension of von Neumann regular rings? [**Lam**]
- When can you create a ring of fractions from a domain?
- Give an example of a von Neumann regular ring which is not boolean. [**Bergman**]
- Suppose a ring R has characteristic m, can it be embedded into a ring of characteristic zero?
- Suppose that R has characteristic m, but does not have a unity. Can R be embedded into a ring of characteristic m with unity?
- Can a non commutative ring free of zero divisors be embedded into a division ring?
- Can the ring of $n \times n$ matrices over a ring R be mapped homomorphically onto the ring of $(n-1) \times (n-1)$ matrices?
- Define group ring. What are the open problems on groups rings? What are the relationships among the open problems? What classes of groups rings can you think of for which the answer is known? [**Bergman**]
- What is Rieffel's Theorem? How does it imply the Wedderburn-Artin theorem? [**Lam**]
- State and prove Nakayama's Lemma. [**Lam**]

- What must the dimension of a (finite-dimensional) division ring over its center be? Construct an example where it has dimension n^2. [**Bergman**]
- Show that any (finite dimensional) central simple algebra over an algebraically closed field splits. [**Lam**]
- Define projective and injective modules. Characterise projectives. Exhibit a non-free projective. [**Lam**]
- What does the Brauer group have to do with group cohomology? [**Lam**]
- State and prove the Wedderburn-Artin theorem. Does every ring have a non-zero semisimple quotient. [**Lam**]
- Let k be a division ring, V be a right k-vector space and $R = \mathrm{End}(V)$. What are the ideals of R? Is R isomorphic to its opposite? [**Bergman**, who admits that the question is difficult, restricts to the case where both k and $\dim V$ are countably infinite and offers several hints]

  What is the Jacobson radical of R? [**Lam**]
- What is a local ring? [**Lam**]

  Exhibit a local subring of the ring of $n \times n$ matrices over a field. [**Bergman**]
- What is the relationship between local rings and indecomposable modules? Exhibit an indecomposable module which is not strongly indecomposable. [**Lam**]
- State the Krull-Schmidt-Azumaya Theorem. Give an example of a module which has two nonequivalent decompositions into finitely many indecomposable modules. [**Lam**]