Tubular Neighborhoods of Nodal Sets

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Abstract

Let (M, g) be a closed compact smooth Riemannian Manifold of dimension n. Let f be an eigenfunction of the Laplace-Beltrami operator on M with eigenvalue λ. $\{f = 0\}$ is called the λ-nodal set.

Yau's conjecture says: The $(n-1)$-dimensional volume of $\{f = 0\}$ is comparable to $C\sqrt{\lambda}$. This conjecture was proved in the case where M is real analytic by H. Donnelly and C. Fefferman.

We consider a tubular neighborhood $T_{\lambda,r}$ of radius r of the λ-nodal set. We show that on real analytic manifolds

$$C_1\sqrt{\lambda}\, r < \mathrm{Vol}(T_{\lambda,r}) < C_2\sqrt{\lambda}\, r \ .$$

This shows a regularity property of the nodal set and may lead to curvature estimates of the nodal set. This is joint work with Dmitry Jakobson.

References

[1] *Tubular Neighborhoods of Nodal Sets and Diophantine Approximation*, arXiv:0707.4045.